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IX.—*On Typical Mountain Ranges: an Application of the Calculus of Probabilities to Physical Geography.* By WILLIAM SPOTTISWOODE, Esq., M.A., F.R.S., F.R.G.S.

Read, April 23rd, 1860.

IN an elaborate memoir published in the 'Petersburg Transactions' (Series VI., tom. viii.), Dr. Abich has, among other things, illustrated the views of A. Von Humboldt and Ritter on the directions of the mountain systems which form the great plateau of Central Asia, by showing that the same views may be extended to the highlands of Western Iran. Grouping the ranges of the Caucasus, of Georgio-Armenia, and of Northern Persia, under four heads, he deduces a mean direction for each group. But, probably laying more stress upon the geological and other parts of his subject than upon these numerical calculations, he has taken only the arithmetical mean of the directions of the ranges under consideration, without reference to either their length or their elevation. I propose here to resume, with some other modifications, the calculation of a mean or typical direction, taking into account (so far as is approximately possible) the length and elevations, or, in other words, the mass of the mountains upheaved. But in so doing, my object is not so much to correct Dr. Abich's conclusions on this particular point,—which are in fact independent of the largest correction furnished by the subjoined calculations,—as to suggest and exemplify a new problem in physical geography.

It is well known that the Calculus of Probabilities enables us to deduce, from a number of discordant observations, the most probable value of the quantity observed, the probable errors of various observations, the "error to be feared" in the probable value, and the limits within which the probable value probably lies, as well as many other results of a like kind. I here regard the minor deviations of a group of mountains lying mainly in the same direction, as "errors" or deviations from a true typical direction; and calculate the other errors, probabilities, and limits as in the case of a group of observations. It is on a similar principle that the Calculus of Probabilities has been applied to questions of gunnery.

The data of the following investigations are as follow: consider the spherical triangle A B C, in which A and B are the two extremities of the range, or portion of range, under consideration, and C the North Pole. The positions of the points A and B being astronomically determined, the side c (opposite to C) is found from the colatitudes of A and B, and their difference of longitude, by the usual formula. The direction of c, and consequently that of the range, is given by the equation

$$\begin{aligned}\theta &= \tfrac{1}{2}(A - 90^\circ + 180^\circ - B - 90^\circ)\\ &= \tfrac{1}{2}(A - B).\end{aligned}$$

The importance of the range A B in determining the typical direction of the group will depend partly upon its length, and partly upon its mass. Now, as we have not before us in detail the breadth and lateral slope of the different ranges or parts of ranges, we shall suppose,—and the supposition is perhaps sufficiently accurate for the present purpose,—that the slope is always the same; in other words, that all transverse sections are similar triangles. This being the case, the mass will always be proportional to $c\,h^2$, where h represents the height; or, if h_1, h_2, be the heights of A and B respectively, it will be approximately proportional to $\frac{c}{4}\,(h_1 + h_2)^2$. This quantity is mathematically called "the weight of the observation," and will be represented by the symbol w.

The following are the formulæ used: let $c_1, c_2, \ldots c_n$, be the values of c for the various ranges of the group; $\theta_1, \theta_2, \ldots \theta_n$, the angles between $c_1, c_2, \ldots c_n$, and parallels of latitude drawn through the middle points of the ranges; $w_1, w_2, \ldots w_n$, the corresponding masses, weights of observations, or values of w. Then the probable typical direction Θ, deduced from the above data, will be given by the formula

$$\Theta \doteq \frac{\Sigma\, w\, \theta}{\Sigma\, w}$$

Moreover, the probable errors, or deviations of the various ranges from the direction of the typical range, will be

$$e_1 = \Theta - \theta_1,\ e_2 = \Theta - \theta_2,\ \ldots\ e_n = \Theta - \theta_n\,.$$

The mean error (deviation) η of an observation (range) whose weight (mass) is unity is

$$\eta = \sqrt{\frac{\Sigma\, w\, e^2}{n-1}}\,.$$

The mean errors of $\theta_1, \theta_2, \ldots \theta_n$, respectively, or the "errors (deviations) to be feared" will be

$$\varepsilon_1 = \frac{\eta}{\sqrt{w_1}},\ \varepsilon_2 = \frac{\eta}{\sqrt{w_2}},\ \ldots\ \varepsilon_n = \frac{\eta}{\sqrt{w_n}},$$

and the mean error E (deviation) of Θ, or "error (deviation) to be feared,"

$$\mathrm{E} = \frac{\eta}{\sqrt{\Sigma\, w}}\,.$$

So that Θ will lie between the limits $\Theta \pm \mathrm{E}$.

From the various groups considered by Dr. Abich I offer only one example, because the others scarcely contain observations (ranges) sufficiently numerous to give valuable results. In this case θ is measured positively from east southwards, from west northwards, and negatively in an opposite direction. The longitudes are east from Ferro. B′ is the exterior angle of which B is the interior. The value of c is given in geographical miles; that of h in feet.

EAST AND WEST RANGES IN GEORGIO-ARMENIA.

Name of Mountain	Latitude.			Longitude.							
	°	′	″	°	′	″	°	′	″		
Gorobani ..	41	52	10	62	20	45	$B'=94$	2	42	$h_1=3,180$	$c=5\cdot8$
Yalno	41	50	46	62	48	23	$A=93$	44	16	$h_2=6,020$	(1)
Nageba	41	50	11	60	34	36	$B'=91$	57	53	$h_1=8,559$	$c=2\cdot79$
Metizchari ..	41	50	33	60	19	37	$A=91$	47	53	$h_2=9,350$	(2)
Tioneti	41	44	50	62	7	58	$B'=90$	22	42	$h_1=6,110$	$c=5\cdot8$
Ardjewan ..	41	44	45	61	36	53	$A=90$	2	0	$h_2=8,810$	(3)
Ljalwar	41	9	16	62	14	37	$B'=88$	20	22	$h_1=8,420$	$c=12\cdot5$
Uch Tälpälar	41	7	30	61	8	15	$A=87$	36	43	$h_2=9,790$	(4)
Margeoz Dagh	40	43	10	62	55	17	$B'=86$	32	17	$h_1=9,980$	$c=7\cdot29$
Tech Achmet	40	41	18	62	16	54	$A=86$	7	15	$h_2=10,230$	(5)
Murow Dagh	40	16	49	64	0	23	$B'=95$	41	18	$h_1=12,290$	$c=4\cdot47$
Kundur Dagh	40	18	33	63	37	3	$A=95$	26	13	$h_2=11,080$	(6)
Mychtiukan	39	53	20	63	45	44	$B'=95$	42	18	$h_1=11,880$	$c=4\cdot76$
Kyrch-Kiss ..	39	53	29	64	10	27	$A=95$	26	27	$h_2=9,350$	(7)
Mychtiukan	39	53	20	63	45	44	$B'=102$	6	19	$h_1=11,880$	$c=11\cdot32$
Goesel-Dara	40	0	46	63	12	10	$A=101$	44	45	$h_2=11,710$	(8)
Ararat	39	42	11	61	57	43	$B'=94$	41	21	$h_1=16,940$	$c=4\cdot8$
Sor Dagh ..	39	43	28	61	36	53	$A=94$	28	3	$h_2=10,660$	(9)
Agri Dagh (1)	40	2	13	60	55	39	$B'=$			$h_1=9,051$	$c=8\cdot78$
Ditto (2)	40	1	35	60	51	45	$A=$			$h_2=9,184$	(10)
Geschtasar ..	38	49	27	64	53	9	$B'=94$	51	11	$h_1=10,310$	$c=7\cdot41$
Kamku Dagh	38	47	14	63	31	13	$A=93$	5	28	$h_2=9,670$	(11)
Sahalan	38	16	9	65	30	3	$B'=98$	8	57	$h_1=15,000$	$c=7\cdot94$
Kaschka Dagh	38	20	32	64	49	58	$A=97$	44	7	$h_2=10,310$	(12)

From these we deduce the following tables; wherein w has been divided throughout by 10,000, which is equivalent to taking the heights to the nearest 100 feet; and $w\ \theta$ is expressed in seconds.

	°	′	″			
$\theta_1=$	3	53	29	$\omega_1=12,272$	$\sqrt{\omega}_1=110\cdot7$	$\omega_1\,\theta_1=171,920,904$
$\theta_2=$	1	52	53	$\omega_2=22,599$	$\sqrt{\omega}_2=150\cdot3$	$\omega_2\,\theta_2=153,063,027$
$\theta_3=$	0	12	21	$\omega_3=32,625$	$\sqrt{\omega}_3=180\cdot6$	$\omega_3\,\theta_3=24,175,055$
$-\theta_4=$	2	1	28	$\omega_4=103,512$	$\sqrt{\omega}_4=321\cdot7$	$-\omega_4\,\theta_4=754,397,000$
$\theta_5=$	3	40	14	$\omega_5=74,365$	$\sqrt{\omega}_5=269\cdot5$	$\omega_5\,\theta_5=982,662,222$
$\theta_6=$	5	33	45	$\omega_6=61,190$	$\sqrt{\omega}_6=247\cdot3$	$\omega_6\,\theta_6=1,225,320,817$
$\theta_7=$	5	35	27	$\omega_7=54,497$	$\sqrt{\omega}_7=233\cdot4$	$\omega_7\,\theta_7=1,096,863,020$
$\theta_8=$	11	55	32	$\omega_8=157,619$	$\sqrt{\omega}_8=397\cdot0$	$\omega_8\,\theta_8=6,766,916,316$
$\theta_9=$	4	34	41	$\omega_9=91,411$	$\sqrt{\omega}_9=392\cdot3$	$\omega_9\,\theta_9=1,506,545,206$
$-\theta_{10}=$	5	56	40	$\omega_{10}=74,314$	$\sqrt{\omega_{10}}=272\cdot2$	$-\omega_{10}\,\theta_{10}=1,590,322,222$
$-\theta_{11}=$	3	58	19	$\omega_{11}=74,100$	$\sqrt{\omega_{11}}=272\cdot2$	$-\omega_{11}\,\theta_{11}=1,039,547,619$
$\theta_{12}=$	7	56	32	$\omega_{12}=128,061$	$\sqrt{\omega_{12}}=357\cdot8$	$\omega_{12}\,\theta_{12}=3,661,358,416$

Whence $\Sigma\,\omega=886{,}565$, $\Sigma\,\omega\,\theta=12{,}204{,}558{,}142$, $\sqrt{\Sigma}\,\omega=941\cdot6$

$\Theta\ =3^\circ\ \ 49'\ \ 26''$.

It may here be noticed that Dr. Abich's arithmetical mean is 2° 26′ 32″, giving a difference of 1° 22′ 54″.

	°	′	″		°	′	″
$e_1 =$	0	4	3	$e_7 =$	1	46	1
$-e_2 =$	1	56	23	$e_8 =$	8	6	6
$-e_3 =$	3	37	5	$e_9 =$	0	45	15
$-e_4 =$	5	50	54	$-e_{10} =$	9	46	6
$-e_5 =$	0	9	12	$-e_{11} =$	7	47	45
$e_6 =$	1	44	19	$e_{12} =$	4	7	6

Whence—

	′	″		′	″
$\eta =$	9	38·4	$\epsilon_7 =$	0	2·5
$\epsilon_1 =$	0	5·2	$\epsilon_8 =$	0	1·4
$\epsilon_2 =$	0	3·8	$\epsilon_9 =$	0	1·9
$\epsilon_3 =$	0	3·2	$\epsilon_{10} =$	0	2·1
$\epsilon_4 =$	0	1·8	$\epsilon_{11} =$	0	2·1
$\epsilon_5 =$	0	2·1	$\epsilon_{12} =$	0	1·6
$\epsilon_6 =$	0	2·3	$E =$	1	1·4

And Θ lies between 3° 50′ 37″·4 and 3° 48′ 34″·6.

In reference to the foregoing investigations, the question will naturally arise, what degree of confidence can we attach to our conclusion of a typical range inclined at an angle 3° 49′ 36″, or at all events somewhere between the limits 3° 50′ 37″·4 and 3° 48′ 34″·6, to the parallel of latitude passing through it? Is there any good reason for supposing that such a range may be taken by the physical geographer as a true representative of the group under consideration? In the instance before us, owing mainly to the small number of observations, and to the severity of the test to which we subject them, the assurance we receive is but moderate in degree; but the test is a necessary one, the assurance is of a positive character, and the investigations at all events will exemplify the problem here suggested. The answer to the above questions is to be found in the following tables.*

In the first table the first column indicates how far the direction of each range deviates from that of the typical range calculated; east-southern deviations being reckoned positive, and east-northern negative. The second column gives the relative “weight” which each range has had in determining the typical direction. The third column gives the corresponding numbers, if the total “weight” has been represented by 1000; so that each number in the third column represents, according to experiment, the number of chances out of a thousand, or the probability which each corresponding variation in the first column has in its favour. The fourth column has been formed by means of the third in the following manner. From the number 500 written at the top of this column there has been deducted the number 178, placed opposite in the third column; and from the remainder, 322, there has been deducted the number 144, placed opposite in the third column; and so on to a remainder 48. A similar process has been applied from the bottom of the column.

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* I have followed a method of Quetelet, and in several places have adopted his own words.

TABLE I.

Difference in Directions by Variations from the Mean.	Weight of Observations.		Probability of the Variations according to Observation.	Rank of preceding Numbers in Scale of Precision.
	Absolute.	Relative.		
″				
292	158	178	·500	..
148	128	144	·322	14·5
64	54	61	·178	7·25
63	61	69	·117	4·75
27	91	103	·048	2
..	..	..	·055	2·20
2	12	13	·068	2·75
− 6	74	84	·152	6·20
− 70	23	26	·178	7·33
−130	33	37	·215	9·
−211	104	117	·332	14·75
−281	74	84	·416	21·75
−352	74	84	·500	..
..	886	1000	..	..

It will be observed that, in performing the subtraction from the positive end, we cannot proceed beyond the remainder 48. This remainder expresses one part of the observations which enter the group 27″; the other part is represented by 55, derived from the negative end. The two together make up 103, as they should. This sum is, in other words, the "weight" of the ranges lying between $27'' \times \frac{1}{2}(63''-27'') = 27''-18''= 45''$, and $27''-\frac{1}{2}(27''-2'') = 27''-12''\cdot5 = 14''\cdot5$. The two parts should theoretically have turned out in the proportions 18 : 12·5. These facts indicate that the mean has been calculated somewhat too low, and that it should have been between 2″ and 27″ higher, but nearer to the latter value than the former. This correction, it should be noticed, lies within the limits of probable error $E = 0^\circ\ 1'\ 1''\cdot4$, calculated above.

In the last column are given the results of comparing the numbers in the third with a "general scale of precision," so as to ascertain the places they occupy. The numbers in a general scale of precision, such as here used, are half the probabilities of showing at least 499,498, &c., and not more than 500,501, &c., balls of a given colour in 999, drawn from an urn containing an infinite number of balls in equal proportions of black and white. Thus ·048, or rather a number very near to it, is found in the second place of such a table, and ·215 in the ninth. The other numbers in the fourth column are found to lie intermediate between the various places in the Table of Precision. It is in this fifth column that we may see whether the directions of the ranges proceed with regularity, and whether they vary from that of the typical range in conformity with the indications of theory. Theoretically the differences of the numbers in this fifth column should be nearly equal, except for the extreme terms; and although this is not strictly the case here, we may still represent the column with tolerable accurary by a double arithmetical progression whose first term is 2, and whose difference is 4·25 in either direction.

Lastly, proceeding on this basis we may form the following Table:—

TABLE II.

Difference in Direction by Variations from the Mean.	Rank acccording to Calculation.	Probability of Variations according to		Weight of Observations.	
		Calculation.	Observation.	By Calculation.	By Observation.
″					
292	..	·500	·500	168	173
148	14·75	·332	·322	86	144
64	10·5	·246	·178	92	61
63	6.25	·154	·117	102	69
27	2	·050	·048	104	103
2	2·25	·057	·068	18	13
− 6	6·5	·160	·152	103	84
− 70 −130	10·75	·252	·215	92	63
−211	15	·339	·332	87	117
−281	19·25	·389	·416	50	84
−352	..	·500	·500	111	84

The agreement of calculation and experience, as exhibited by a comparison of the third column of this table with the fourth, and of the fifth with the sixth, although not perfect, is yet sufficiently approximate to justify the conclusion that, the various mountain ranges which we have been considering do point to a common type, that their directions are not accidental, and that the geologist and the physical philosopher will at least have good grounds for seeking some common agency which has caused their upheaval.

In this way the Calculus of Probabilities, though one of the most abstract and refined branches of mathematics, and in itself incapable of interpreting any natural phenomena, may still serve as a check and a guide to the physical philosopher, by pointing out where he may and where he may not employ his study of causes with reasonable hope of a successful result.